

Hgame-2019几道有趣的RSA

原创

[Gard3nia](#) 于 2019-03-03 15:28:12 发布 841 收藏 1

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9 篇文章 0 订阅

订阅专栏

babyRSA

```
e = 12
p = 58380004430307803367806996460773123603790305789098384488952056206615768274527
q = 81859526975720060649380098193671612801200505029127076539457680155487669622867
ciphertext =
20608721532369020246787892668194449176965915672645869081591928616363088644729157051019617158562614360898838
4615185921752409380788006476576337410136447460
算出的m转化成字符串
```

说实话这题本来以为很简单的, 想直接求私钥, 直接求解, 没想到算出来 $\varphi(n)$ 和 $e=12$ 不互素, 无法求出 e 对应的公钥, 所以直接解是没有办法的;

解法如下:

$$m^e \equiv C \pmod{n}$$

$$m^{12} \equiv C \pmod{n} \quad (\varphi(n), 12) \neq 1$$

无法求 e^{-1}

分解 $e = 3 \times \varphi$

$$(m^4)^3 \equiv C \pmod{n} \quad \because (\varphi(n), 3) = 1$$

\therefore 求出 3 的逆元 d

$$\text{则 } m^4 \equiv C^d \pmod{n}$$

$$m^4 = \left\{ \begin{array}{l} (m^2)^2 \quad \text{2 次 Rabin} \\ \text{或 直接开 4 次方} \end{array} \right.$$

解题脚本如下：

```

#coding:utf-8
import gmpy2
import math
def rabin_decrypt(c, p, q, e=2):
    n = p * q
    mp = pow(c, (p + 1) / 4, p)
    mq = pow(c, (q + 1) / 4, q)
    yp = gmpy2.invert(p, q)
    yq = gmpy2.invert(q, p)
    r = (yp * p * mq + yq * q * mp) % n
    rr = n - r
    s = (yp * p * mq - yq * q * mp) % n
    ss = n - s
    return (r, rr, s, ss)

def main():
    e = 12
    p = 58380004430307803367806996460773123603790305789098384488952056206615768274527
    q = 81859526975720060649380098193671612801200505029127076539457680155487669622867
    ciphertext = 20608721532369020246787892668194449176965915672645869081591928616363088644729157051019617158562614
3608988384615185921752409380788006476576337410136447460
    n=p*q
    ol=(p-1)*(q-1)
    d=gmpy2.invert(3,ol)
    ciphertext1=pow(ciphertext,d,n)
    ans=rabin_decrypt(ciphertext1,p,q,e=2)
    for i in ans:
        temp=rabin_decrypt(i,p,q,e=2)
        for x in temp:
            flag='{:x}'.format(x).decode('hex')
            if 'hgame' in flag:
                print flag

if __name__ == '__main__':
    main()

```

easy_rsa

m为17位十进制数，提交格式hgame{m}

e1:0x33240

e2:0x3e4f

n:0x9439682bf1b4ab48c43c524778c579cc844b60872275725c1dc893b5bcb358b9f136e4dab2a06318bb0c80e202a14bc54ea334519bec023934e01e9378abf329893f3870979e9f2f2be8fff4df931216a77007a2509f49f697bf286285e97fac5dc6e4a164b5c2cc430887b18136437ba67777bda05aafdeaf918221c812b4c7d1665238f84ab0fab7a77fcae92a0596e58343be7a8e6e75a5017c63a67eb11964970659cd6110e9ec6502288e9e443d86229ef2364dfecb63e2d90993a75356854eb874797340eece1b19974e86bee07019610467d44ec595e04af02b574a97fa98bdb2e779871c804219cab715f4a80fef7f8fb52251d86077560b39c1c2a1

c1:0x7c7f315a3ebbe305c1ad8bd2f73b1bb8e300912b6b8ba1b331ac2419d3da5a9a605fd62915c11f8921c450525d2efda7d48f1e503041498f4f0676760b43c770ff2968bd942c7ef95e401dd7facbd4e5404a0ed3ad96ae505f87c4e12439a2da636f047d84b1256c0e363f63373732cbaf24bda22d931d001dcca124f5a19f9e28608ebd90161e728b782eb67deeba4cc81b6df4e7ee29a156f51a0e5148618c6e81c31a91036c982debd1897e6f3c1e5e248789c933a4bf30d0721a18ab8708d827858b77c1a020764550a7fe2ebd48b6848d9c4d211fd853b7a02a859fa0c72160675d832c94e0e43355363a2166b3d41b8137100c18841e34ff52786867d

c2:0xf3a8b9b739196ba270c8896bd3806e9907fca2592d28385ef24afadc2a408b7942214dad5b9e14808ab988fb15fbd93e725edcc0509ab0dd1656557019ae93c38031d2a7c84895ee3da1150eda04cd2815ee3debaa7c2651b62639f785f6cabf83f93bf3cce7778ab369631ea6145438c3cd4d93d6f2759be3cc187651a33b3cc4c3b477604477143c32dff62461dfd9f8aa879257489bbf977417ce0f8e89e3f2464475624aafef57dd9ea60339793c69b53ca71d745d626f45e6a7beb9fcb9d1a259433d36139345b7bb4f392e78f1b5be0d2c56ad50767ee851fac670946356b3c05d0605bf243b89c7e683cc75030b71633632fb95c84075201352d6

c1=pow(m, e1, n)

c2=pow(m, e2, n)

先转化为10进制，发现e1=209472，e2=15951，e1,e2不互素，最大公因数为3；本想用共模攻击，但是e1和e2不互素，没办法达到共模攻击的条件，但是仔细推来发现：

$\gcd(e1, e2) = 3$ ，则 $\gcd(e1/3, e2/3) = 1$

所以令 $a1 = e1/3, a2 = e2/3$ ，则 $\gcd(a1, a2) = 1$

所以存在 $r \cdot a1 + s \cdot a2 = 1$ ，所以 $r \cdot e1 + s \cdot e2 = 3$ (扩展欧几里得算法)

则 $(c1^r) \cdot (c2^s) = (m^{r \cdot e1}) \cdot (m^{s \cdot e2}) = m^{r \cdot e1 + s \cdot e2} = m^3$ //共模攻击

所以到此为止： $(m^3) \bmod n = (c1^r) \cdot (c2^s) \bmod n$ 为已知数

所以直接低指数攻击即可

上解题脚本：

```

#coding:utf-8
import gmpy2
def same_attack(e1,e2,mess1,mess2,n):
    gcd,s,t=gmpy2.gcdext(e1,e2)
    if s<0:
        s=-s
        mess1=gmpy2.invert(mess1,n)
    if t<0:
        t=-t
        mess2=gmpy2.invert(mess2,n)
    mess=gmpy2.powmod(mess1,s,n)*gmpy2.powmod(mess2,t,n)%n
    return mess

def small_attack(c,n,e):
    i=0
    while 1:
        if(gmpy2.iroot(c+i*n, e)[1] == 1):
            x = gmpy2.iroot(c+i*n, e)[0]
            print x
            break
        i += 1

def main():
    e1="0x33240"
    e2="0x3e4f"
    n="0x9439682bf1b4ab48c43c524778c579cc844b60872275725c1dc893b5bcb358b9f136e4dab2a06318bb0c80e202a14bc54ea334519b
ec023934e01e9378abf329893f3870979e9f2f2be8fff4df931216a77007a2509f49f697bf286285e97fac5dc6e4a164b5c2cc430887b181
36437ba67777bda05aafdeaf918221c812b4c7d1665238f84ab0fab7a77fcae92a0596e58343be7a8e6e75a5017c63a67eb11964970659cd
6110e9ec6502288e9e443d86229ef2364dfecb63e2d90993a75356854eb874797340eece1b19974e86bee07019610467d44ec595e04af02b
574a97fa98bdb2e779871c804219cab715f4a80fef7f8fb52251d86077560b39c1c2a1"
    c1="0x7c7f315a3ebbe305c1ad8bd2f73b1bb8e300912b6b8ba1b331ac2419d3da5a9a605fd62915c11f8921c450525d2efda7d48f1e503
041498f4f0676760b43c770ff2968bd942c7ef95e401dd7facbd4e5404a0ed3ad96ae505f87c4e12439a2da636f047d84b1256c0e363f633
73732cbaf24bda22d931d001dcca124f5a19f9e28608ebd90161e728b782eb67deeba4cc81b6df4e7ee29a156f51a0e5148618c6e81c31a9
1036c982debd1897e6f3c1e5e248789c933a4bf30d0721a18ab8708d827858b77c1a020764550a7fe2ebd48b6848d9c4d211fd853b7a02a8
59fa0c72160675d832c94e0e43355363a2166b3d41b8137100c18841e34ff52786867d"
    c2="0xf3a8b9b739196ba270c8896bd3806e9907fca2592d28385ef24afadc2a408b7942214dad5b9e14808ab988fb15fbd93e725edcc05
09ab0dd1656557019ae93c38031d2a7c84895ee3da1150eda04cd2815ee3debaa7c2651b62639f785f6cabf83f93bf3cce7778ab369631ea
6145438c3cd4d93d6f2759be3cc187651a33b3cc4c3b477604477143c32dff62461fdfd9f8aa879257489bbf977417ce0f8e89e3f246447
5624aafef57dd9ea60339793c69b53ca71d745d626f45e6a7beb9fcbd9d1a259433d36139345b7bb4f392e78f1b5be0d2c56ad50767ee851
fac670946356b3c05d0605bf243b89c7e683cc75030b71633632fb95c84075201352d6"
    e1=int(e1,16)/3
    e2=int(e2,16)/3
    n=int(n,16)
    c1=int(c1,16)
    c2=int(c2,16)
    # print gmpy2.gcd(e1,e2)
    ans=same_attack(e1,e2,c1,c2,n)
    # print ans
    small_attack(ans,n,3)

if __name__ == '__main__':
    main()

```

加上“hgame”即可得到flag:

```
59594981651654789  
[Finished in 0.2s]
```