BUUCTF笔记之Crypto部分WriteUp（一）

原创

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分类专栏： Crypto CTF 文章标签： crypto 信息安全

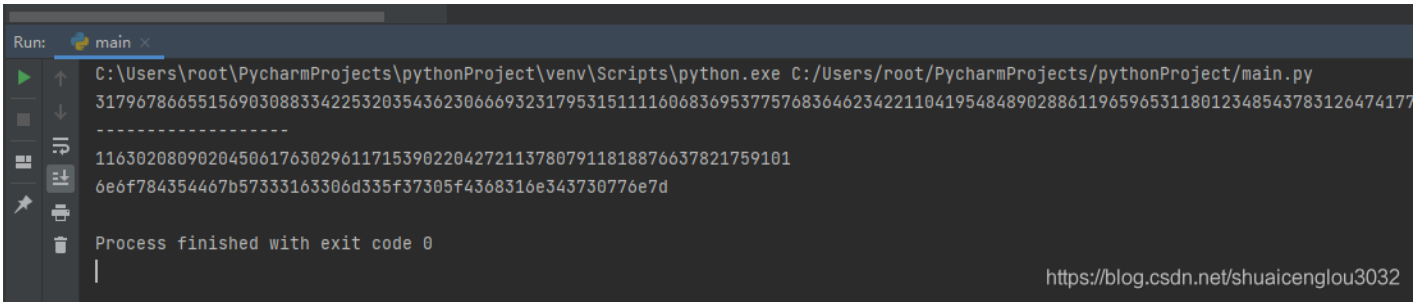
1、RSA1

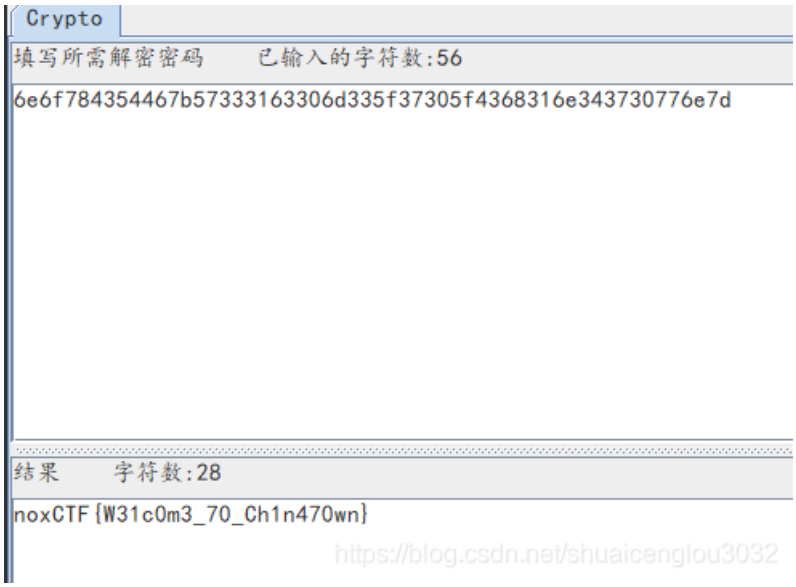
直接上代码：

```
import gmpy2 as gp

p = 8637633767257008567099653486541091171320491509433615447539162437911244175885667806398411790524083553445
q = 1264067497399647276917604793717088342092705082148001058159313713537247388059561373733763062975257734614
dp = 6500795702216834621109042351193261530650043841056252930930949663358625016881832840728066026150264693307
dq = 7834722636735534490195325803864706723805740335513038891379117604388816836745560980982567957673512201963
c = 2472230540388738207356731646764908066263155290596022939907910799560215441817605633580063888752761416407
n = p*q
phin = (p-1)*(q-1)
dd = gp.gcd(p-1, q-1)
d=(dp-dq)//dd * gp.invert((q-1)//dd, (p-1)//dd) * (q-1) +dq
print(d)
m = gp.powmod(c, d, n)
print('------------------')
print(m)
print(hex(m)[2:])
```

拿到十六进制的明文之后要转换成字符串：



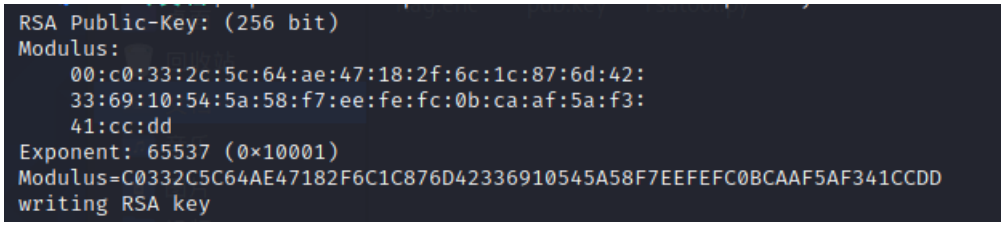


这里转换成字符串之后提交，不对。我又在外面加了一层flag{}，也不对，最后灵机一动，把nnxCTF换成了flag，成功了。

2、RSA

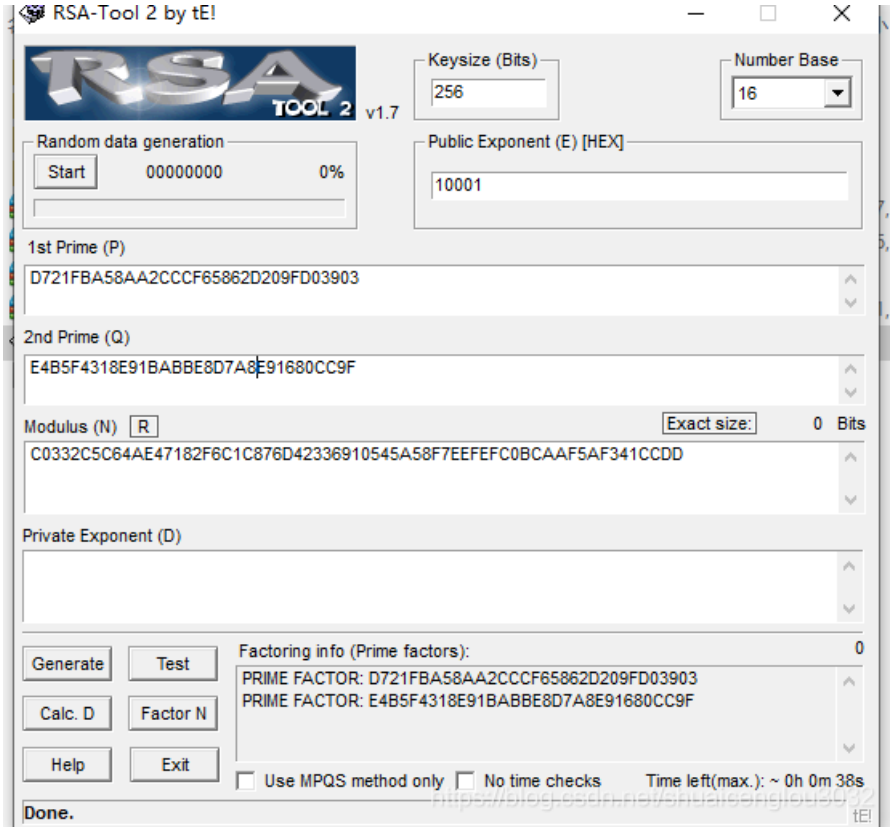
解析RSA公钥：

openssl rsa -pubin -text -modulus -in pub.key:



拿到N为：C0332C5C64AE47182F6C1C876D42336910545A58F7EEFEFC0BCAAF5AF341CCDD

爆破一下（这个比较耗时间，我R7的机器用了半个多小时才算出来，所以还是直接在线爆破吧）：



得到了p和q，然后上面可以看到e是65537.

所以就可以生成私钥并解密了：

python rsatool.py -f PEM -o key.key -p 1 -q 1 -e 1

openssl rsautl -decrypt -inkey key.pem -in flag.enc -out flag

```
tom@kali:~/文档$ python3 rsatool.py -f PEM -o key.key -p 285960468890451637935629440372639283459 -q 30400874160460192449432815597527241846 3 -e 65537
Using (p, q) to initialise RSA instance

n =
c0332c5c64ae47182f6c1c876d42336910545a58f7eefefc0bcaaf5af341ccdd

e = 65537 (0x10001)

d =
b378155840fb2b8fbdd869db5b7e91994f1ece256ee1175ec2c2bd3a4a795ad1

p = 285960468890451637935629440372639283459 (0xd721fba58aa2cccf65862d209fd03903)

q = 30400874160460192449432815597527241846 3 (0xe4b5f4318e91babbe8d7a8e91680cc9f)

Saving PEM as key.key
```


然后解密（这里记得把生成的key.key改成key.pem）：

```
tom@kali:~/文档$ ls
flag.enc  key.key  pub.key  rsatool.py
tom@kali:~/文档$ openssl rsautl -decrypt -inkey key.pem -in flag.enc -out flag
Can't open key.pem for reading, No such file or directory
140482968364352:error:02001002:system library:fopen:No such file or directory:../crypto/bio/bss
_file.c:69:fopen('key.pem','r')
140482968364352:error:2006D080:BIO routines:BIO_new_file:no such file:../crypto/bio/bss_file.c:
76:
unable to load Private Key
tom@kali:~/文档$ mv key.key key.pem
tom@kali:~/文档$ ls
flag.enc  key.pem  pub.key  rsatool.py
tom@kali:~/文档$ openssl rsautl -decrypt -inkey key.pem -in flag.enc -out flag
tom@kali:~/文档$ ls
flag  flag.enc  key.pem  pub.key  rsatool.py
tom@kali:~/文档$ cat flag
flag{decrypt_256}
tom@kali:~/文档$
```


3、[NPUCTF2020]共 模 攻 击

先上hint的解密代码

```
from gmpy2 import *
from libnum import *
from sympy import *
p = 10731697577128434210836295494509648970890030263373452094390528365528331853570 9
n = 6807492006219935335233722232024809784434293293172317282814978688931711423939629682224374870233587969960
e1 = 2303413961
c1 = 1754421169036191391717309256938035960912941109206872374826444526733030696056821731708193270151759843 78
e2 = 2622163991
c2 = 1613454015951555289711148366977297613624544025937559371784736059448454437652633847111272619248126613 50
s = gcdext(e1,e2)
c = pow(c1,s[1],n)*pow(c2,s[2],n) % n
print(c)
m = nthroot_mod(c,256,p)
print (hex(m))
print(n2s(m))
```

得到明文：

m.bit_length() < 400

有点懵逼。看大佬的wp才知道是Coppersmith定理攻击。

4、[NCTF2019]easyRSA

直接上代码：

```
from gmpy2 import *
from Crypto.Util.number import *
from tqdm import tqdm
def onemod(p,r):
    t = p-2
    while pow(t, (p-1) // r, p) == 1: t -= 1
    return pow(t, (p-1) // r, p)
def solution(p,root):
    g = onemod(p, 0x1337)
    may = []
    for i in range(0x1337):
        if i % 100 == 0:
            print(i)
        may.append(root * pow(g,i,p) % p)
    return may
c = 10562302690541901187975815594605242014385201583329309191736952454310803387032252007244962585846519762059
p = 19913867782374383733992752015760782002974657455774654909492148829287722650919831501601891938525978123819
q = 11221369590547214241522144451532653232035242947834168335281118350326967655543460122901367931942387823899
e = 0x1337
n = p * q
c_p = pow(c, (1 + 3307 * (p - 1) // e) // e, p)
C1 = solution(p, c_p)
c_q = pow(c, (1 + (2649 * (q - 1) // e)) // e, q)
C2 = solution(q, c_q)
a = invert(p, q)
b = invert(q, p)
flag=0
for i in tqdm(C1):
    for j in tqdm(C2):
        flag += 1
        if flag % 1000000 == 0:
            print(flag)
        x = (b * q * i + a * p * j) % n
        print(x)
        if b"NCTF{" in long_to_bytes(x):
            print(long_to_bytes(x))
            exit()
```

这道题非常难，可以说是数学的交锋了，看了wp才知道简直是有毒，直接上出题人的wp了（原链接在这里）：

我们知道，RSA对参数的一个要求就是，e和phi(n)一定要互素。这是为了要让e在模phi(n)下存在逆元d，进而可以直接`pow(c, d, n)`来解密。

那如果e和phi(n)不互质就会无解么？不，事实上，**有解而且不止有一解**。

这一题就是基于这个观察而出的。

题面十分简洁，甚至都给出了`p, q`。乍一看，肯定觉得这是一道**送分题**，然而事实远非如此。

正常情况下的RSA都要求`e`和`phi(n)`要互素，不过也有一些`e`和`phi(n)`有很小的公约数的题目，这些题目基本都能通过计算`e`对`phi(n)`的逆元`d`来求解。

然而本题则为e和p-1(或q-1)的最大公约数就是e本身，也就是说e | p-1，只有对c开e次方根才行。
可以将同余方程
$$
m^e \equiv c \quad (\text{mod}\ n)
$$
化成
$$
\begin{aligned}
m^e &\equiv c \quad (\text{mod}\ p)\newline
m^e &\equiv c \quad (\text{mod}\ q)
\end{aligned}
$$
然后分别在GF(p)和GF(q)上对c开e=0x1337次方根，再用CRT组合一下即可得到在mod n下的解。

问题是，**如何在有限域内开根？**

这里e与p-1和q-1都不互素，不能简单地求个逆元就完事。

这种情况下，**开平方根**可以用Tonelli-Shanks algorithm，wiki说这个算法可以**扩展到开n次方根**。

在这篇paper里给出了具体的算法：Adleman-Manders-Miller rth Root Extraction Method

> 应该还有其他的算法。。不过这一个对我来说比较容易去implement。

Table 4: Adleman-Manders-Miller rth root extraction algorithm in F_q

Input: F_q and a rth residue δ, $r|q-1$.
Output: A rth root of δ.

Step 1: Choose ρ uniformly at random from F_q^*.
Step 2: **if** $\rho^{\frac{q-1}{r}} = 1$, **go to** Step 1.
Step 3: Compute t, s such that $q-1 = r^t s$, where $(r, s) = 1$.
Compute the least nonnegative integer α such that $s|r\alpha - 1$.
Compute $a \leftarrow \rho^{r^{t-1}s}, b \leftarrow \delta^{r\alpha-1}, c \leftarrow \rho^s, h \leftarrow 1$
Step 4: **for** $i = 1$ **to** $t-1$
compute $d = b^{r^{t-1-i}}$
if $d = 1$, $j \leftarrow 0$,
else $j \leftarrow -\log_a d$ (compute the discrete logarithm)
$b \leftarrow b(c^r)^j$, $h \leftarrow h c^j$
$c \leftarrow c^r$
end for
Step 5: **return** $\delta^\alpha \cdot h$

这个算法只能开出一个根，实际上开0x1337次方，最多会有0x1337个根（这题的情况下有0x1337个根）。

如何找到其他根？
StackOverflow – Cube root modulo P给出了方法：

2 This code provides *one* of the three cube roots. How to get the other two? – Shebla Tsama Jan 17 '18 at 22:31

2 @SheblaTsama multiply by a primitive cube root of 1. – deinst Jan 18 '18 at 20:46

1 @Denis Yes. In most cases when p is 1 mod 3 there is no cube root. – deinst Jun 21 '18 at 12:31

如何找到所有的`primitve 0x1337th root of 1`？
StackExchange – Finding the n-th root of unity in a finite field给出了方法：

6

In a finite field of size q, the multiplicative subgroup has order $q-1$ (i.e. all elements are invertible except 0). If n is relatively prime to $q-1$, then there is only one n-th root of unity, i.e. 1 itself. If n divides $q-1$, then there are n roots of unity. In the remainder of this answer, I assume that you are in that case, i.e. n divides $q-1$.

Everything I write below uses computations *in the finite field* (i.e. modulo q, if q is prime).

To get an n-th root of unity, you generate a random non-zero x in the field. Then:

$$(x^{(q-1)/n})^n = x^{q-1} = 1$$

Therefore $x^{(q-1)/n}$ is an n-th root of unity. Note that you can end up with any of the n n-th roots of unity (including 1 itself), each with probability $1/n$.

Now you may want to have a *primitive* n-th root of unity, i.e. one value g such that all n-th roots of unity can be obtained with values g^j for integers j ranging from 0 to $n-1$. If g is an n-th root of unity, then it is primitive if and only if $g^j \neq 1$ for any $j > 0$ that divides n, except n itself. In your case this is easy: since n is a power of 2, any j that divides n is also a power of 2. In practice, this yields the following:

Exploit

- 先用`Adleman-Manders-Miller rth Root Extraction Method`在`GF(p)`和`GF(q)`上对`c`开`e`次根，分别得到一个解。大概不到10秒。
- 然后去找到所有的`0x1336`个`primitive nth root of 1`，乘以上面那个解，得到所有的`0x1337`个解。大概1分钟。
- 再用`CRT`对`GF(p)`和`GF(q)`上的两组`0x1337`个解组合成`mod n`下的解，可以得到`0x1337**2==24196561`个`mod n`的解。最后能通过`check`的即为`flag`。大概十几分钟。
- `exp.sage`如下：

```
import random
import time

# About 3 seconds to run
def AMM(o, r, q):
    start = time.time()
    print('\n--------------------------------------------------------------------------------')
    print('Start to run Adleman-Manders-Miller Root Extraction Method')
    print('Try to find one {:#x}th root of {} modulo {}'.format(r, o, q))
    g = GF(q)
    o = g(o)
    p = g(random.randint(1, q))
    while p ^ ((q-1) // r) == 1:
        p = g(random.randint(1, q))
    print('[+] Find p:{}'.format(p))
    t = 0
    s = q - 1
    while s % r == 0:
        t += 1
        s = s // r
```

```
    print('[+] Find s:{}, t:{}'.format(s, t))
    k = 1
    while (k * s + 1) % r != 0:
        k += 1
    alp = (k * s + 1) // r
    print('[+] Find alp:{}'.format(alp))
    a = p ^ (r**(t-1) * s)
    b = o ^ (r*alp - 1)
    c = p ^ s
    h = 1
    for i in range(1, t):
        d = b ^ (r^(t-1-i))
        if d == 1:
            j = 0
        else:
            print('[+] Calculating DLP...')
            j = - dicreat_log(a, d)
            print('[+] Finish DLP...')
        b = b * (c^r)^j
        h = h * c^j
        c = c ^ r
    result = o^alp * h
    end = time.time()
    print("Finished in {} seconds.".format(end - start))
    print('Find one solution: {}'.format(result))
    return result

def findAllPRoot(p, e):
    print("Start to find all the Primitive {:#x}th root of 1 modulo {}.".format(e, p))
    start = time.time()
    proot = set()
    while len(proot) < e:
        proot.add(pow(random.randint(2, p-1), (p-1)//e, p))
    end = time.time()
    print("Finished in {} seconds.".format(end - start))
    return proot

def findAllSolutions(mp, proot, cp, p):
    print("Start to find all the {:#x}th root of {} modulo {}.".format(e, cp, p))
    start = time.time()
    all_mp = set()
    for root in proot:
        mp2 = mp * root % p
        assert(pow(mp2, e, p) == cp)
        all_mp.add(mp2)
    end = time.time()
    print("Finished in {} seconds.".format(end - start))
    return all_mp


c = 10562302690541901187975815594605242014385201583329309191736952454310803387032252007244962585846519
p = 19913867782374383733992752015760782002974657455774654909492148829287722650919831501601891938525978
q = 11221369590547214241522144451532653232035242947834168335281118350326967655543460122901367931942387
e = 0x1337
cp = c % p
cq = c % q
mp = AMM(cp, e, p)
mq = AMM(cq, e, q)
p_proot = findAllPRoot(p, e)
```

```
q_proot = findAllPRoot(q, e)
mps = findAllSolutions(mp, p_proot, cp, p)
mqs = findAllSolutions(mq, q_proot, cq, q)
print mps, mqs

def check(m):
    h = m.hex()
    if len(h) & 1:
        return False
    if h.decode('hex').startswith('NCTF'):
        print(h.decode('hex'))
        return True
    else:
        return False


# About 16 mins to run 0x1337^2 == 24196561 times CRT
start = time.time()
print('Start CRT...')
for mpp in mps:
    for mqq in mqs:
        solution = CRT_list([int(mpp), int(mqq)], [p, q])
        if check(solution):
            print(solution)
    print(time.time() - start)

end = time.time()
print("Finished in {} seconds.".format(end - start))
```

Summary

p, q都是预先用下面这个函数生成的，保证了e | p-1, e | q-1。

```
import random
from Crypto.Util.number import *

def gen():
    p = e * random.getrandbits(1012) + 1
    while not isPrime(p):
        p = e * random.getrandbits(1012) + 1
    return p
```

而且p-1, q-1的ord(e) = 1，使得Adleman-Manders-Miller rth Root Extraction Method中无需计算DLP。降低了题目难度。

flag后面填充了一段杂乱的字符串，是为了增加flag转成整数后的位数。不然位数太低，算出GF(p)和GF(q)里2组0x1337个解，取交集就可以得到flag了。位数增加后，就必须要算24196561次CRT才能得到flag，可能需要个十几分钟来跑。

----------无话可说，给出题人跪下了----------

5、丢失的MD5

我还以为要写代码穷举，结果一条命令搞定

```
python md5.py
```

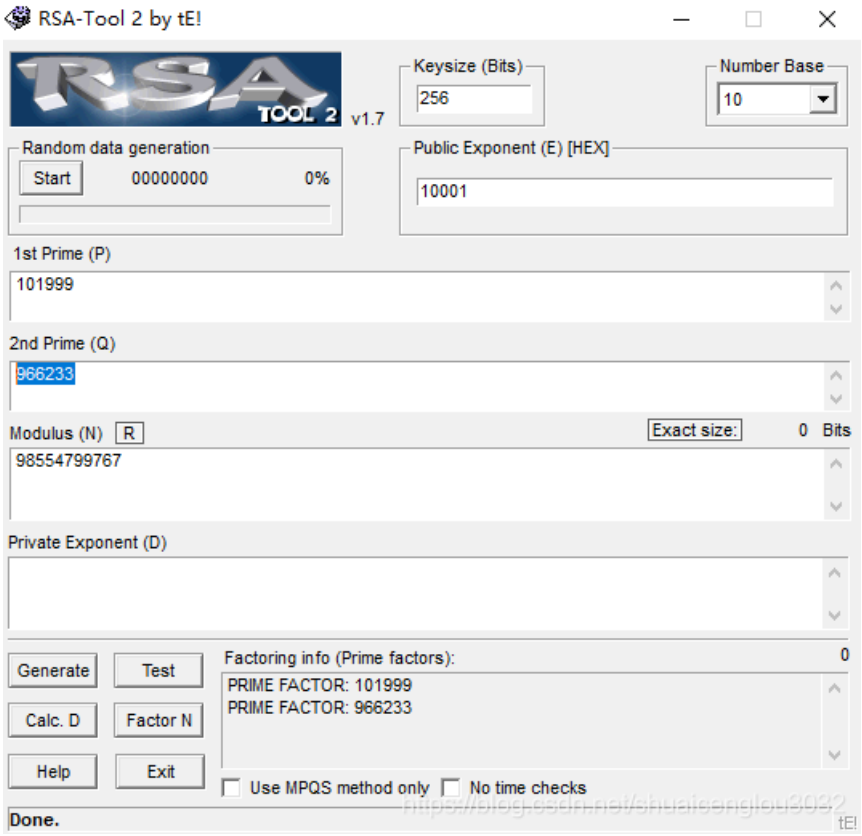
```
tom@kali:~/文档$ python
Python 2.7.18 (default, Mar  9 2021, 11:09:26)
[GCC 10.2.1 20210110] on linux2
Type "help", "copyright", "credits" or "license" for more information.
>>> exit()
tom@kali:~/文档$ python md5.py
e9032994dabac08080091151380478a2
tom@kali:~/文档$
```

加上flag就是答案。

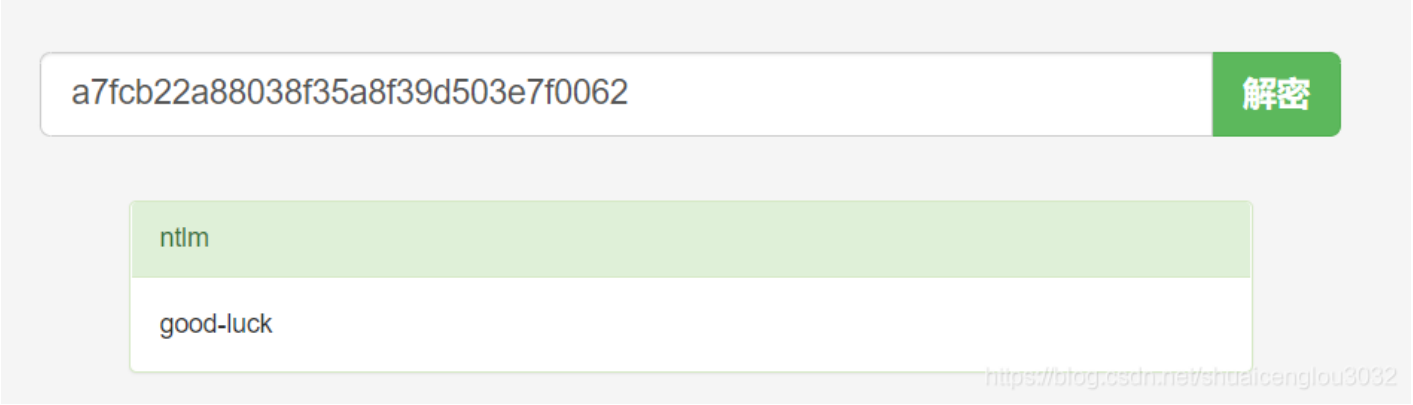
6、Alice与Bob

密码学历史中，有两位知名的杰出人物，Alice和Bob。他们的爱情经过置换和轮加密也难以混淆，即使是没有身份认证也可以知根知底。就像在数学王国中的素数一样，孤傲又热情。下面是一个大整数:98554799767,请分解为两个素数，分解后，小的放前面，大的放后面，合成一个新的数字，进行md5的32位小写哈希，提交答案。
注意：得到的 flag 请包上 flag{} 提交

粗暴：



7、Windows系统密码



8、传统知识+古典密码

将题目中的内容和天干地支的顺序对应，得出一数字字符：28 30 23 8 17 10 16 30，但题目中还提到“信的背面还写有“+甲子”，”，然后各加六十（一甲子代表60年），变成 88 90 83 68 77 70 76 90；与ASII码一一对应得 XZSDMFLZ，然后再来凯撒一波，得到flag{SHUANGYU}

9、传感器

55555555595555A65556AA696AA6666666955
这是某压力传感器无线数据包解调后但未解码的报文(hex)

已知其ID为0xFED31F，请继续将报文完整解码，提交hex。

提示1：曼联

根据提示及大佬的wp知道是曼彻斯特编码

先转二进制：

```
10101010101010101010101010101011001010101010101010110100110010101010101011010101010011101
```

IEEE 802.4（令牌总线）和低速版的IEEE 802.3（以太网）中规定, 按照这样的说法, 01电平跳变表示1, 10的电平跳变表示0。

然后每两位根据上述规则转为0或者1，再转十六进制即可，此处给出大佬的代码：

```
cipher='5555555595555A65556AA696AA6666666955'
def iee(cipher):
    tmp=''
    for i in range(len(cipher)):
        a=bin(eval('0x'+cipher[i]))[2:].zfill(4)
        tmp=tmp+a[1]+a[3]
        print(tmp)
    plain=[hex(int(tmp[i:i+8][::-1],2))[2:] for i in range(0,len(tmp),8)]
    print(''.join(plain).upper())

iee(cipher)
```

运行就能拿到flag。

9.1、传感器1

这里另外给出一道差分曼彻斯特编码的题：

已知ID为0x8893CA58的温度传感器的未解码报文为：3EAAAAA56A69AA55A95995A569AA95565556
此时有另一个相同型号的传感器，其未解码报文为： 3EAAAAA56A69AA556A965A5999596AA95656

解码该报文得到flag。

先把已知ID的报文转二进制：

11111010101010101010101010010101101010011010011010101001010101101010010101100110010101101

这题是差分曼彻斯特编码，和上面第九题稍有不同，识别差分曼彻斯特编码的方法：主要看两个相邻的波形，如果后一个波形和前一个的波形相同，则后一个波形表示0，如果波形不同，则表示1。

因此上面的二进制四个一组的看，一组中前两个等于后两个则转换成0，不同转换成1：

1111 1010

上代码：

```
a = 0x3EAAAAA56A69AA55A95995A569AA95565556
b = 0x3EAAAAA56A69AA556A965A5999596AA95656
b2 = bin(b)
print len(b2)
str = ""
for i in range(len(b2[2:]) / 2):

    a1 = b2[i * 2:i * 2 + 2]
    a2 = b2[i * 2 + 2:i * 2 + 4]

    if a2 != '10' and a2 != '01':
        continue
    if a1 != '10' and a1 != '01':
        continue
    if a1 != a2:
        str += '1'
    else:
        str += '0'
print len(str)
print hex(int(str, 2)).upper()
# 0X10024D8893CA584181L
# 0X30024D8845ABF34119L
```

10、信息化时代的步伐

http://code.mcdvisa.com/

中文电码反查汉字结果：

- 6060：计
- 4615：算
- 2623：机
- 6008：要
- 1783：从
- 1216：娃
- 1216：娃
- 2119：抓
- 6386：起

本地工具：