

# 【CTF WriteUp】2020电信和互联网行业赛个人赛部分

## Crypto题解

原创

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### Crypto

(话说题目做一半就当答案是什么鬼)

### Crypto-bacon

#### 题目

```
flag{AAAABAAAAAABAABBBABABABAAABAABAAAABBAABAAABABBABAAAAABAABAAAABBBABABAABAABA}
```

#### 解答

简单的培根密码，略

### Crypto-黄金分割RSA

#### 题目

encryption

```
[1, 28657, 2, 1, 3, 17711, 5, 8, 13, 21, 46368, 75025, 34, 55, 89, 610, 377, 144, 233, 1597, 2584, 4181, 6765, 10946, 987]
```

output

```
publickey=[0x1d42aea2879f2e44dea5a13ae3465277b06749ce9059fd8b7b4b560cd861f99144d0775ffffffffffff, 5]  
c=421363015174981309103786520626603807427915973516427836319727073378790974986429057810159449046489151
```

#### 解答

(本题为2020 GACTF-da Vinci after rsa，然后河南天安杯又考一遍，这是第三遍)



```

#!/usr/bin/env python
# -*- coding: utf-8 -*-
from Crypto.Util.number import *
import gmpy2

def GCRT(mi, ai):
    assert (isinstance(mi, list) and isinstance(ai, list))
    curm, cura = mi[0], ai[0]
    for (m, a) in zip(mi[1:], ai[1:]):
        d = gmpy2.gcd(curm, m)
        c = a - cura
        assert (c % d == 0) # 不成立则不存在解
        K = c / d * gmpy2.invert(curm / d, m / d)
        cura += curm * K
        curm = curm * m / d
    return (cura % curm, curm) # (解, 最小公倍数)

n = 0x1d42aea2879f2e44dea5a13ae3465277b06749ce9059fd8b7b4b560cd861f99144d0775ffffffffffff
c = 421363015174981309103786520626603807427915973516427836319727073378790974986429057810159449046489151
p = 9749
q = 11237753507624591
r = n / p / q
e = 5

p_roots = [7361]
q_roots = [2722510300825886, 6139772527803903, 6537111956662153, 8415400986072042, 9898464751509789]
r_roots = [180966415225632465120208272366108475667934082405238808958048294287011243645, 281611441149332825868287
3357893989007684496552202823306045771363205185148674391, 1369135259891793292334345751773139388112378132927363770
631732500241630990458667, 5570877862584063114417410584640901580756179707042774516590562822938385811269597, 84990
52407588078002885931765166137308397074232361087682974448633946350539292222]

m_list = []
for pp in p_roots:
    for qq in q_roots:
        for rr in r_roots:
            res = GCRT([p, q, r], [pp, qq, rr])[0]
            if pow(res, e, n) == c:
                print long_to_bytes(res)

```

得到一个字符串flag{weadfa9987\_adwd23123\_454f}, 但是我们还有一个条件没有使用, 即一串数

```
[1, 28657, 2, 1, 3, 17711, 5, 8, 13, 21, 46368, 75025, 34, 55, 89, 610, 377, 144, 233, 1597, 2584, 4181, 6765, 10946, 987]
```

仔细观察发现，这些数就是斐波那契数列的一个排列，且其总数为25，恰好与flag{}内的字符数相同，因此将字符按照同样方式打乱后可以得到真实flag。

	A	B	C	D	E
1	w	1		1	w
2	e	1		28657	5
3	a	2		2	a
4	d	3		1	e
5	f	5		3	d
6	a	8		17711	4
7	9	13		5	f
8	9	21		8	a
9	8	34		13	9
10	7	55		21	9
11	-	89		46368	4
12	a	144		75025	f
13	d	233		34	8
14	w	377		55	7
15	d	610		89	-
16	2	987		610	d
17	3	1597		377	w
18	1	2584		144	a
19	2	4181		233	d
20	3	6765		1597	3
21	-	10946		2584	1
22	4	17711		4181	2
23	5	28657		6765	3
24	4	46368		10946	-
25	f	75025		987	2
26					

(不要问我为什么原题需要按加密方式打乱而不是按照加密方式进行恢复，问GACTF出题人)

## Crypto-Corrupted Keys

### 题目

ciphertext.txt

```
c = 0x6f9c3479883b414030032610a0831089ea2d5f6598d16f8b3415dbb7ff88e6214c7704dbaf1f0f0fe8243468b203b0c128933ab45f406109d234ab94457aa4ff81de3e0c1dda55b95344683e7cfe4e39dedd1203120af89e14702ac54a1a21adb500dad67033deb2dcf844aa10c5b6425aca0a756ee5e5ce5b583de68d7dfa675b8142c4b175b347bd1c3b2d2cd32aa2e03356ecf4821704d7b7542a22d09ebb239e382fc5b72ea051b65596e41d228fb7b0f7acf5686d05b8d6807a26c1a1d92c8b116c6f27e2b21ded5f1f3b8f9a88e45ec7b14aee18e74454fefb1a482a9eafc9550d16f6683e2f7cbd0d9ce9a474f4db01e2f97d0d3d23fad566489e1e
```

private\_key.pem

```
-----BEGIN RSA PRIVATE KEY-----
MIIEowIBAAKCAQEAAANAL4ECQAIAsACAUJBfAA0NIAADdwAIAGAAAAALoAAAAABw
PwBQAAAAAnwAAcP4NAAE3QFAAcJMKwJbGANcAADcJdHc7DgAAANIAMaOAXgAAMQAA
AOAAIAYQIFFEJAAgCYA8GwAHgAw1KcFhAAHzwAKAAAAA2BgMAYEBZIDcnAAEMMAAE
QcAAUVFQAD3VEABAsQAAAAAMAegilfw0A8ALWABBAIKAAAAAIQAwApwkArfyDwG
ALAAAQA9FBAoBqJAKAAgKUABzOQawlnoGAgUMYAoAAAAACAoCAHowABAAwUHAJCy
AgUFEA4AAAAA0AIAOAOAAjwDADQDUeAAKIA7DAAIDAQABAoIBAAAAAAAAAAAAADgAA
APvgAAEAAAAAAAAAAAAACgAAhAAAAABgAAAJAAAAAAAAAAAAAADAAAA0AAAAA4A4A
AAAAAGAAAAAAEAAAAAAAAAAAAAAACwAAgAAAAAIAAAAA0AAAAAAABAAACBAA
AA8AAAAABgAAKAAAAAANAABBgAAAAAAwAAIAAAAAADgAAAAAAAAAAAAAACAACAAAA
AAAAAAAAAACgAAAAAAAAAAAAABgABAAAAAsAACAgAAAAAAAAAAAAABAAAAwCACQ
AAAOAN0AAAEHAAAAAAAAAAAAAAAAAAAAcAAAAAAAAAAAAQAAAAAAAAAAAA
AAFAAACgYEAAAAAAAAQAAAAAAAAAAAAFAAAAAgAMAcACgAKAAAArAAAAAAAA
AAAAAAAAAACQADAAAAAAAAAAAAACwAAAAcWAAgAAAAAAAAANAAAAAAAAAAAAAAAg
AAAAAAAAAMGAAAAAAAAAAAsAAAAQACAAADAAAAAAAAAAAAFAAAAAAAAAAACgYEAACw
AKAAAAA0AAAEAAHAAAAAAAAAAAAAPQAAAAAAAAAAAAACAAAA4AALMAAAAABQAAoAAA
AAAAAAAAABQAAAAAKAOAAACAAAAAAAwIAAAAAA4MAAAAAAAAGQAAGAAAAAA
AAADAACAAAJAAAAUUAAAAAAAAAAAAAAAAAAAAcGYAFOW7M5UcyswjtKNXo783B
hdUPHTPG49nzxsU33eLi8pxZ6hFaFPaE08NBkBHMqPI61Pn/wuisNqVWna5igQEA
AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAG0Fr7+S3oAktME9GMkvPYiqk1qeDYLZX
1mkrXy1v2aJ74q+J41UdYwKBgAAg8QsAAACgBQCQAAAAAAAAAAgAAqQAABgDQAAAA
AAAAAAAAAcAoAAAABAawAAAawBgAAsw0gCAAAAAAAAAAGAAAAA8AAAAAAAEwAAAA
AACgAAAAAAGAAAAAAACVAAUAAAAADgAAAAAAAAkAAAgAAKAAAAAALAAAAADQ
5AAAAoGBAAAAAAAOAUA0AwCDwAABADJAAAAAHAAAAAADgAAAAAAAAAAAAAI8A
AAYPAMAAADAAAAIAAAAAACQAAGAAAOAcAAAAAAAAAAAAAAAAADAAAAAA0AAAAAZ
```

public\_key.pem

```
-----BEGIN PUBLIC KEY-----
MIIBIjANBgkqhkiG9w0BAQEFAAOCAQ8AMIIBCgKCAQEAJdPL4kqVHoXVu7GAWpZf
5y0NKjCDdw4aWkRnGBLrly+fI56P8hcFI3hn6Bfff4dkiE3Q18MeZMaxpbGI9cU
zjqjXhi7Dg8gJtLrORoNXt4PMa9dF+P/L3YRIFH5RhgSa69WwSHTg91KelhLV3zy
VqhtWf421kNAaEJZJjcnaAFcPLM0QcBQUVfXfj3VGcpBsVdcPmMMMRepilf63I+b
LwCRLKqqeVHx4SgxFp0nZrFyDw2croQFFSv9F5AprqJMqMh6VzZz0Wa01nq2gv
U8Z1ohOU7C8oezHoymxDIyVHDZCyAgUFFU6ciic0AYA+QeCCj6nF/fLUeJEaI27D
8QIDAQAB
-----END PUBLIC KEY-----
```

### 解答

这道题本身设置了两个考点：一是如何从残缺的私钥中提出信息，二是如何利用中间被挖去一段的dp来分解n。前者参见前两年Ocutf的一道题，后者参见今年11月辽宁祥云杯。但是这道题私钥直接给全了，导致使用openssl命令直接可以提出数据，第一个考点直接失效。应该如同上边一样给出。

开始解体。首先从公钥中提出n和e

openssl rsa -in public\_key.pem -pubin -modulus -text

```
C:\windows\system32\cmd.exe
C:\Users\Chainer\Desktop
C:\Users\Chainer\Desktop>openssl rsa -in public_key.pem -pubin -modulus -text
RSA Public-Key: (2048 bit)
Modulus:
00:8d:d3:eb:e2:4a:95:1e:85:d5:bb:b1:80:5a:96:
5f:e7:2d:0d:2a:30:83:77:05:78:69:09:11:9c:60:
4b:ae:56:2e:7c:9e:7a:3f:8e:5c:14:8d:e1:9f:a0:
5f:7d:fe:1d:92:21:37:43:5f:0c:79:93:1a:c6:96:
c6:23:d7:14:ce:3a:a3:5e:18:bb:0e:0f:20:26:d2:
eb:39:1a:0d:5e:de:0f:31:af:5d:17:e3:ff:2f:76:
11:20:51:f9:46:18:12:6b:af:56:c1:21:ed:83:dd:
4a:7a:58:4b:57:7c:f2:56:a8:6d:59:fe:36:96:43:
40:69:42:59:26:37:27:68:01:5e:3c:b3:84:41:c0:
50:51:57:af:7e:3d:a5:19:ca:41:b1:87:5e:3e:83:
0c:31:17:a9:8a:57:fa:dc:8f:9e:2d:67:11:24:b2:
aa:a9:e5:47:c7:84:a0:c4:5a:74:9f:3a:df:c8:3c:
36:72:ba:10:14:54:af:f4:5e:40:a5:1a:89:32:ac:
0c:87:a5:73:67:33:96:0b:49:67:ab:68:24:53:c6:
65:a2:13:94:ec:2f:23:7b:31:e8:ca:6c:43:23:25:
47:0d:90:b2:02:05:05:15:4e:9c:8a:27:34:01:80:
3e:41:e0:82:8f:a9:c5:fd:f2:d4:7b:91:1a:23:6e:
ca:ff
Exponent: 65537 (0x10001)
Modulus=8D03EBE24A951E85D5BBB1805A9657E2D002A30837705786969119C604BA5563E7C3E7A3FC85C1480E19FA05F7DFE1D922137435F0C7993
1AC696C623D7147E3AA35E18BB0B0F202ED2E8391A005E0B0F31AF5D17E3FF2F76112051F94618126BAF56C121ED830D4A7A584B577CF256A8D509E
389643498425926372768015C3C33441C0505157D77E3DD519CA41B1575C3E630C3117A9A57FADC8F962D671124B2AAA0E547C784A0C45A749F3A
DFC83C3672BA10145A9F45E40A51A8932A0C87A5736733966B4967AB682F53C65A213945C2F287B31E3CA6C43232547D90B2020505154B9CA27
3401803E41B0828FA9C5FDF2D478911A236E3F1
writing RSA key
https://blog.csdn.net/cccchhhh6819
```

但是仅有n、e、c是无法做题的，因为n不能直接分解，所以我们还需要看看私钥给我们留下了什么。根据资料，我们得知RSA的私钥通常以PKCS#1的模式进行存储，简单地如下所示：

```
RSAPrivateKey ::= SEQUENCE {
    version          Version,
    modulus          INTEGER, -- n
    publicExponent   INTEGER, -- e
    privateExponent  INTEGER, -- d
    prime1           INTEGER, -- p
    prime2           INTEGER, -- q
    exponent1        INTEGER, -- d mod (p-1)
    exponent2        INTEGER, -- d mod (q-1)
    coefficient       INTEGER, -- (inverse of q) mod p
    otherPrimeInfos  OtherPrimeInfos OPTIONAL
}
```

我们将现在已经被污染的私钥base64解码后，按照上边的格式展开，得到如下内容：

```

308204a3020100
0282010100
(n)00d00be04090008000b0008050905f000d0d20000377000800600000000ba00000000703f00500000009f000070fe0d0001374050007
0930ac096c600d7000030a30e10bb0e00000d200300a005e000031000000e000200610205109000802600f06c001e0030d4a705840007cf
000a00000003606030060405920372700010c30000441c0005157d0003dd5100040b100000000c0007a08a57f0d00f002d600104020a000
000008400c00a70900adfc83c0600b000000400f45040a01a8900a00080a5000733906b0967a0602050c600a00000020287001e8c000400
305070090b2020505100e0000003400800e00e0008f00c00d00d47800a200ec300
0203
(e)010001
02820100
(d)0000000000000000e000000f000001000000000000000000000000a000084000000060000009000000000000000c000000d00000
000e00e0000000006000000000040000000000000000000000000b00008000000020000000d000000000001000081000000f000
0000006000090000000d0000000006000000000c00008000000000e00000000000000030000800000000000000000000a000000
0000000000000060001000000000b0000200080000000000000000400000307000900000e00dd0000010700000000000000000000
0000000000000070000000000000001000000000000000000000000050000
02818100
(p)0000000001000000000000000005000000008003007000a000a00000ac0000000000000000000000000090003000000000
00000000000b00000000b0000800000000000d00000000000000000000000020000000000c006000000000000b0000001000200000
30000000000000005000000000000000
02818100
(q)0000b000a00000000d0000010000700000000000000f40000000000000000200000e0000b30000000050000a00000000000
00000000500000000a00e000020000000000000c0800000000e003000000000006400006000000000030000800000900000
05005000000000000000000000000000
028180
(dp)05396ecce54732b308ed28d5e8efcdc184350f1d33c6e3d9f3c6c537dde2e2f29c59ea115a14f6843bc3419011cca8f23a94f9ffc2e8
ac350bd69dae6281010000000000000000000000000000000000000000000000000000083416befe4b7a0092d304f46324bcf622aa4d6a78360
b657d6692b5f2d6fd9a27be2af89e3551d63
028180
(dq)0020f10b000000a00500900000000000020000a900000600d0000000000000000700a0000001000c00000030060000b30d2008
0000000000080000000f0000000000040f000000000a0000000000600000000009500050000000e00000000000900000
8000a0000000000b00000000d0e40000
02818100
(coeff)000000000a00500d00c020f00000400c900000000700000000000e000000000000000008f0000060f00c0000030000020
000000090000800000a00700000000000000000000000000000000c00000000d0000000019 (残缺) 0

```

可以看到，除了dp以外其他内容基本已经无法使用，dp只缺少其中的200位，因此可以尝试使用coppersmith方法去解。由于coppersmith方法需要未知变量系数为1，我们这里尝试推导一下：

我们知道

$$dp * e = k * (p - 1) + 1$$

由于dp <= p-1，所以k < e，coppersmith方法通过遍历1~e寻找k。现在假设k已知，那么有

$$dp * e + k - 1 = 0 \pmod{p}$$

设

$$dp = s_1 * 2^{520} + x * 2^{320} + s_2$$

带入原式得

$$(\varepsilon_1 * 2^{520} + x * 2^{320} + \varepsilon_2) * e + k - 1 = 0 \pmod{p}$$

$$\varepsilon_1 * 2^{520} + x * 2^{320} + \varepsilon_2 + (k - 1) * \text{inv}(e, n) = 0 \pmod{p}$$

$$\varepsilon_1 * 2^{520} * \text{inv}(2^{320}, n) + x + \varepsilon_2 * \text{inv}(2^{320}, n) + (k - 1) * \text{inv}(e, n) * \text{inv}(2^{320}, n) = 0 \pmod{p}$$

这样就构造出了系数为1的同余方程，可以使用coppersmith方法解题了。解题脚本如下：



```

from sage.all import *
from Crypto.Util.number import long_to_bytes

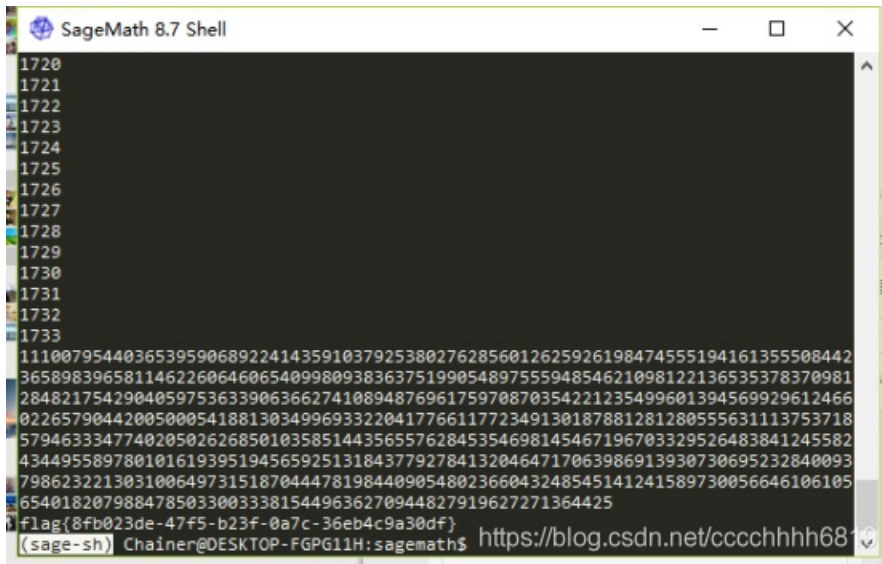
n = 0x8dd3cbe24a951e85d5bbb1805a965fe72d0d2a30837705786969119c604bae563e7c8e7a3fc85c148de19fa05f7dfe1d922137435f
0c79931ac696c623d714ce3aa35e18bb0e0f2026d2eb391a0d5ede0f31af5d17e3ff2f76112051f94618126baf56c121ed83dd4a7a584b57
7cf256a86d59fe3696434068425926372768015c3cb33441c0505157d77e3dd519ca41b1575c3e630c3117a98a57fad8f9b2d671124b2aa
a9e547c784a0c45a749f3adfc83c3672ba101454aff45e40a51a8932ac0c87a5736733966b4967ab682f53c665a21394ec2f287b31e8ca6c
432325470d90b2020505154e9c8a273401803e41e0828fa9c5fdf2d478911a236ec3f1
e = 0x10001
c = 0x6f9c3479883b414030032610a0831089ea2d5f6598d16f8b3415dbb7ff88e6214c7704dbaf1f0f0fe8243468b203b0c128933ab45f
406109d234ab94457aa4ff81de3e0c1dda55b95344683e7cfe4e39dedd1203120af89e14702ac54a1a21adb500dad67033deb2dcf844aa
10c5b6425aca0a756ee5e5ce5b583de68d7dfa675b8142c4b175b347bd1c3b2d2cd32aa2e03356ecf4821704d7b7542a22d09ebb239e382f
c5b72ea051b65596e41d228f7b0f7ac5f5686d05b8d6807a26c1a1d92c8b116c6f27e2b21ded5f1f3b8f9a88e45ec7b14aee18e74454fefb
1a482a9eafc9550d16f6683e2f7cbd0d9ce9a474f4db01e2f97d0d3d23fad566489e1e
s1 = 0x5396ecce54732b308ed28d5e8efcdc184350f1d33c6e3d9f3c6c537dde2e2f29c59ea115a14f6843bc3419011cca8f23a94f9ffc2
e8ac350bd69dae628101
s2 = 0x83416befe4b7a0092d304f46324bcf622aa4d6a78360b657d6692b5f2d6fd9a27be2af89e3551d63
invE = 607246348005222277448444018894068784893261339674776416517053926300280106832311997118643400122356432947960
3123138991852830747459341281298547464945329258352716978884436175214634055317578153997659440696241077620973159496
0444089673635388970465490762413236424460315237883100650480470692160869167806717582595930549357426298413851694659
3961101500800318974736058847108299210208951476938115767279186343649800059109625400737095596094353444034846045686
1334522885919455014210659853726203177705141103038754371809891816964608801118434914236230355247754146610175756211
785759304159230507837819605093784920450130362742047348692105621
invpow = 1116443150207074751150053002174551090668533485688766572101159798693207919778016196338123839488045291036
2678853293436976134225721195690734856639343780389327693213031686657799035703254166242337351340806493525437022390
0703723931988947315436601244964166984345494128713782600950862786002028205007904344114955555693973765288323596742
9254789631975028782111078451934403051031125206688037746177755118530359200184630061799358019685860767135508139753
8545831351265009506760347727474227285668988817907365295771630760269871647111667154970535637451277321913843646237
831493800721780046762737829851933071451680344164843758329746830379

def coppersmith(k):
    F.<x> = PolynomialRing(Zmod(n))
    f = (s1 << 520) * invpow + x + s2 * invpow + (k - 1) * invE * invpow # make monic
    x0 = f.small_roots(X=2 ** 200, beta=0.44, epsilon=1/32)
    return x0

for k in range(1, e):
    print k
    x0 = coppersmith(k)
    if len(x0) != 0:
        x = Integer(x0[0])
        dp = (s1 << 520) + (x << 320) + s2
        p = (e*dp - 1) // k + 1
        if p != -1:
            q = n // p
            assert n == p * q
            phi = (p-1)*(q-1)
            d = inverse_mod(e,phi)
            print d
            print long_to_bytes(pow(c,d,n))
            break

```

其中invE、invpow分别是计算好的invert(e, n)和invert(2 \*\* 320,n)，最终求出k=1733。



```
SageMath 8.7 Shell
1720
1721
1722
1723
1724
1725
1726
1727
1728
1729
1730
1731
1732
1733
11100795440365395906892241435910379253802762856012625926198474555194161355508442
36589839658114622606460654099809383637519905489755594854621098122136535378370981
28482175429040597536339063662741089487696175970870354221235499601394569929612466
02265790442005000541881303499693322041776611772349130187881281280555631113753718
57946333477402050262685010358514435655762845354698145467196703329526483841245582
43449558978010161939519456592513184377927841320464717063986913930730695232840093
79862322130310064973151870444781984409054802366043248545141241589730056646106105
654018207988478503300333815449636270944827919627271364425
flag{8fb023de-47f5-b23f-0a7c-36eb4c9a30df}
(sage-sh) Chainer@DESKTOP-FGPG11H: sagemath$ https://blog.csdn.net/ccchhhh681
```

## Crypto-strange\_GSW

### 题目

strange\_GSW.sage

```
from Crypto.Util.number import *
from random import randrange
from hashlib import md5
from secret import FLAG

def GenerateG(_n, _q):
    _len = int(round(log(_q, 2)))
    _G = Matrix(ZZ, _len * _n, _n)
    for i in range(_len):
        for j in range(_n):
            _G[j * _len + i, j] = 2 ** i
    return _G

def BinaryExpansion(_C, _q):
    expansion_C = Matrix(ZZ, _C.nrows(), _C.nrows())
    log_q = _C.nrows() // _C.ncols()
    for i in range(_C.nrows()):
        for j in range(_C.ncols()):
            bits = bin(_C[i, j] % _q)[2:].rjust(log_q, '0')[::-1]
            for index in range(log_q):
                expansion_C[i, j * log_q + index] = int(bits[index])

    return expansion_C

def KeyGen(degree, _q, _B):
    s = random_matrix(ZZ, degree, 1, x=_q // 4, y=4 * _q // 3)
    return Matrix(s.list() + [-1]).transpose(), s

def BitEncrypt(plain_bit, _n, _q, _B, _G, _s):
```

```

_m = int(round(log(_q, 2)) * _n)
A = random_matrix(ZZ, _m, _n - 1, x=0, y=_q)
e = random_matrix(ZZ, _m, 1, x=0, y=_B)
_C = block_matrix([A, A * _s + e], ncols=2, subdivide=False) + plain_bit * _G
_C = BinaryExpansion(_C, _q)
return _C

def Encrypt(plain, _n, _q, _B, _G, _s):
    plain_bits = bin(plain)[2:]
    Cipher = []
    for bit in plain_bits:
        Cipher.append(BitEncrypt(int(bit), _n, _q, _B, _G, _s))
    return Cipher

if __name__ == '__main__':
    n = 11
    q = 65537
    B = q // 256
    G = GenerateG(n, q)
    decrypt_key, encrypt_key = KeyGen(n - 1, q, B)
    flag = FLAG + md5(long_to_bytes(randrange(2 ** 127, 2 ** 128))).hexdigest().encode()[:8] + b}'

    _cipher = Encrypt(bytes_to_long(flag), n, q, B, G, encrypt_key)
    str_cipher = ' '.join([str(i.list()) for i in _cipher])
    with open('flag', 'wb') as f:
        f.write(flag)
    with open('cipher', 'w') as f:
        f.write(str_cipher)

```

## 解答

(比赛好像没人做出来，确实非常费劲)

题目看起来比较复杂，所以我们跟着程序走，看看具体流程是什么样的。首先看矩阵G

```
SageMath 8.7 Shell
(sage-sh) Chainer@DESKTOP-FGPG11H:sagemath$ sage test.sage
[ 1 0 0 0 0 0 0 0 0 0 0]
[ 2 0 0 0 0 0 0 0 0 0 0]
[ 4 0 0 0 0 0 0 0 0 0 0]
[ 8 0 0 0 0 0 0 0 0 0 0]
[16 0 0 0 0 0 0 0 0 0 0]
[32 0 0 0 0 0 0 0 0 0 0]
[64 0 0 0 0 0 0 0 0 0 0]
[128 0 0 0 0 0 0 0 0 0 0]
[256 0 0 0 0 0 0 0 0 0 0]
[512 0 0 0 0 0 0 0 0 0 0]
[1024 0 0 0 0 0 0 0 0 0 0]
[2048 0 0 0 0 0 0 0 0 0 0]
[4096 0 0 0 0 0 0 0 0 0 0]
[8192 0 0 0 0 0 0 0 0 0 0]
[16384 0 0 0 0 0 0 0 0 0 0]
[32768 0 0 0 0 0 0 0 0 0 0]
[ 0 1 0 0 0 0 0 0 0 0 0]
[ 0 2 0 0 0 0 0 0 0 0 0]
[ 0 4 0 0 0 0 0 0 0 0 0]
[ 0 8 0 0 0 0 0 0 0 0 0]
[ 0 16 0 0 0 0 0 0 0 0 0]
[ 0 32 0 0 0 0 0 0 0 0 0]
[ 0 64 0 0 0 0 0 0 0 0 0]
[ 0 128 0 0 0 0 0 0 0 0 0]
[ 0 256 0 0 0 0 0 0 0 0 0]
[ 0 512 0 0 0 0 0 0 0 0 0]
[ 0 1024 0 0 0 0 0 0 0 0 0]
[ 0 2048 0 0 0 0 0 0 0 0 0]
[ 0 4096 0 0 0 0 0 0 0 0 0]
[ 0 8192 0 0 0 0 0 0 0 0 0]
[ 0 16384 0 0 0 0 0 0 0 0 0]
[ 0 32768 0 0 0 0 0 0 0 0 0]
[ 0 0 1 0 0 0 0 0 0 0 0]
[ 0 0 2 0 0 0 0 0 0 0 0]
[ 0 0 4 0 0 0 0 0 0 0 0]
[ 0 0 8 0 0 0 0 0 0 0 0]
[ 0 0 16 0 0 0 0 0 0 0 0]
[ 0 0 32 0 0 0 0 0 0 0 0]
```

可知矩阵G是一个固定的矩阵。然后生成一组加密密钥和解密密钥看看

```
(sage-sh) Chainer@DESKTOP-FGPG11H:sagemath$ sage test.sage
[67705]
[27731]
[60070]
[84178]
[76070]
[40469]
[43933]
[23247]
[32280]
[29827]
+++++
[67705]
[27731]
[60070]
[84178]
[76070]
[40469]
[43933]
[23247]
[32280]
[29827]
[ -1]
```

可以看到加密密钥为一个10 \* 1的矩阵，其中每项的值在(q/4, q\*4/3)之间，q = 65537。解密密钥就比加密密钥多一个-1。

最后看加密过程，首先将密文转为整数，再转为二进制，然后对于二进制表示的每一位（0或1）进行BitEncrypt加密操作，将这个0或者1加密成一个176 \* 176的由0和1构造成的矩阵。加密的过程中参数生成随机，所以相同明文加密结果不固定。加密生成的\_C矩阵为一个176 \* 11的矩阵，再经过BinaryExpansion函数扩展到一个176 \* 176的矩阵。这个扩展很简单，就是把这些数每一个按照其二进制表示横向展开成16位，然后倒序写入，因此我们可以简单将其收起得到\_C。

```
def recoverMatrix(text):
    text = text.replace(',', '').replace(' ', '').replace('[', '')
    Mdata = []
    for i in range(0, len(text), 16):
        Mdata.append(int(text[i:i+16][::-1], 2))
    _C = Matrix(ZZ, 176, 11, Mdata)
    return _C
```

在得到`_C`之后，我们来尝试求解key。`_C`的结构是矩阵A拼接一列 $A \cdot s + e$ 的结果，其中 $e$ 是0~256以内的随机值，再根据明文该位置是0还是1来决定是否加上矩阵`_G`。考虑到大部分参数取值范围在 $(q/4, q \cdot 4/3)$ 之间，因此 $e$ 的 $(0, 256)$ 区间可以视为小值，可以将本题当作一个格上的最近向量问题（CVP, Closet vector problem）进行求解。对明文首位进行求解时，针对结果可能为1和0的两种情况，求出的`_C`矩阵需要减去`_G`或者不发生变化。当存在解时，首位明文正确，且求解出的key就是加密所用key，后续每个明文字符直接用该key还原即可。完整解题代码如下：

```
from sage.modules.free_module_integer import IntegerLattice
from Crypto.Util.number import long_to_bytes

def GenerateG(_n, _q):
    _len = int(round(log(_q, 2)))
    _G = Matrix(ZZ, _len * _n, _n)
    for i in range(_len):
        for j in range(_n):
            _G[j * _len + i, j] = 2 ** i
    return _G

def recoverMatrix(text):
    text = text.replace(',', '').replace(' ', '').replace('[', '')
    Mdata = []
    for i in range(0, len(text), 16):
        Mdata.append(int(text[i:i+16][::-1], 2))
    _C = Matrix(ZZ, 176, 11, Mdata)
    return _C

def CVP(lattice, target):
    gram = lattice.gram_schmidt()[0]
    t = target
    for i in reversed(range(lattice.nrows())):
        c = ((t * gram[i]) / (gram[i] * gram[i])).round()
        t -= lattice[i] * c
    return target - t

def BitDecrypt(_C, _G, _s, _q):
    _C = _C * _G * _s
    return 0 if abs(_C.list()[0] % _q - _q) < _q // 4 else 1

def Decrypt(Cipher, _G, _s, _q):
    plain_bits = ''
    for cipher in Cipher:
        plain_bits += str(BitDecrypt(cipher, _G, _s, _q))
    return int(plain_bits, 2)

if __name__ == '__main__':
    n = 11
    q = 65537
    B = q // 256
    G = GenerateG(n, q)

    with open('cipher', 'r') as f:
        cipher_str = f.read()
```

```

cipher_str = cipher_str.split(' ')[:-1]

# Store Ciphertext
_cipher_str = [eval(i.strip()+')]') for i in cipher_str]
_cipher_matrix = [Matrix(176, 176, i) for i in _cipher_str]

# Recover Matrix _C
_C = recoverMatrix(str(cipher_str[0]))
_C = _C - G
_A = _C[:176, :10]
res = _C[:176, 10:].list()

# Make a Matrix for CVP
M = Matrix(ZZ, 186, 176)
for i in range(176):
    for j in range(10):
        M[176 + j, i] = int(_A[i][j])
    M[i, i] = 65537

lattice = IntegerLattice(M, lll_reduce=True)
target = vector(ZZ, res)
res = CVP(lattice.reduced_basis, target)

# Recover Key
R = IntegerModRing(65537)
M = Matrix(R, _A[:176])
key = M.solve_right(res)
key = [int(i) for i in key]
print key

# get flag
flag = long_to_bytes(Decrypt(_cipher_matrix, G, Matrix(key + [-1]).transpose(), q))
print(flag)

```

```

SageMath 8.7 Shell
Starting subshell with Sage environment variables set. Don't forget
to exit when you are done. Beware:
* Do not do anything with other copies of Sage on your system.
* Do not use this for installing Sage packages using "sage -i" or for
running "make" at Sage's root directory. These should be done
outside the Sage shell.
Bypassing shell configuration files...
Note: SAGE_ROOT=/opt/sagemath-8.7
(sage-sh) Chainer@DESKTOP-FGPG11H:~$ cd sagemath
(sage-sh) Chainer@DESKTOP-FGPG11H:sagemath$ sage test.sage
[30111, 35709, 54736, 16648, 17862, 62630, 63649, 19403, 20553, 28790]
flag{13_G3W_r3a1ly_fu0?_th13_ju3t_@_e@3y_ch@11-dd9140cc}
(sage-sh) Chainer@DESKTOP-FGPG11H:sagemath$

```

<https://blog.csdn.net/cccchhhh6810>

## Crypto-random\_fault

(我真不会国密。。。)